Galois Cohomology Seminar
Spring Semester 2019

Joshua Ruiter

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1 Group cohomology

Week 1, Joshua. Group rings, equivalent definitions of group cohomology, long exact sequence, example computations.
[2] 3.1, 3.2 [3] 2.1 [5] 1.1, 1.2, 1.3, 1.10

Week 2, Nick. Inflation and restriction maps, compatible pairs, inflation-restriction exact sequence. Shapiro's lemma, coinduced modules.
[2] 3.3 [3] 2.1 [5] 1.8

Week 3, Joshua. Dimension shifting, cup products for group cohomology.
[2] 3.3 , 3.4 [3] 2.1 (page 60), 2.2 [5] 1.5, 1.7, 1.8

Week 4, Stan. Group homology, Tate cohomology, cohomological triviality for induced/coinduced modules, Tate cohomology for cyclic groups, Herbrand quotients.
[3] 2.3 [5] 1.4, 1.6, 1.1 2 [1] Lecture 23

Week 5, Stan. Tate's theorem, direct/inverse limits, topological and profinite groups, cohomology of profinite groups and discrete modules.
[2] Chapter 1 appendix A, 4.1, 4.2 [3] 2.3, 2.4 [5] 1.4, 1.6, 1.12, 2.1, 2.2

Week 6, Joshua. Galois cohomology, Hilbert theorem 90 in terms of Galois cohomology, Galois correspondence for infinite extensions, absolute Galois group, computation of Brauer group of a finite field.
[2] 4.3 [3] 2.1 (page 65) , Chapter 1 appendix A [5] 2.3, 2.4

2 Brauer groups

Week 7, Joshua. Central simple algebras, Wedderburn's thoerem, definition of Brauer group in terms of central simple algebras, various aspects of well-defined-ness for Brauer group multiplication.
[2] 2.1-2.4 [3] 4.1 [4] 1.1

Week 8, Nick. Splitting fields, Skolem-Noether theorem, relative Brauer groups, maximal subfields of central simple algebras.
[2] 2.1-2.4 [3] 4.2 [4] 1.2, 1.3

Week 9, Nick and Joshua. (Nick) Double centralizer theorem, maximal subfields, absolute Brauer group as union of relative Brauer groups for finite Galois extensions. (Joshua) Applications of Skolem-Noether and maximal subfield results to $\mathrm{Br}(\mathbb{R})$ and $\mathrm{Br}(\mathbb{F}_q)$.
[2] 2.1-2.4 [3] 4.2 [4] 1.2, 1.3

Week 10, Joshua. Relationship between cohomological Brauer group and CSA Brauer group, factor sets. Possibly cyclic algebras.
[3] 4.3 (especially 3.16) [4] 1.4

Week 11, Joshua. Brauer group of a local field, invariant maps of local class field theory. [4] Section 6

3 Local class field theory

Week 12, Stan. Local Artin map.

Week 13, Stan. More on the local Artin map.

References

[1] Mit math 18.785 - number theory 1 lecture notes. Available at http://math.mit.edu/classes/18.785/2018fa/lectures.htm.

[2] Philippe Gille and Tamás Szamuely. Central simple algebras and group cohomology, 2006.

[3] J.S. Milne. Class field theory (v4.02), 2013. Available at www.jmilne.org/math/.

[4] Igor Rapinchuk. The brauer group of a field. Available at https://drive.google.com/file/d/0B0CCc00SqXL4dTBIbU8xa0Vjb2c/edit.

[5] Romyar Sharifi. Group and galois cohomology. Available at http://math.ucla.edu/ sharifi/groupcoh.pdf.